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Simulating the Movement of Planets in the Solar System Using a Linear System

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Abstract: This article discusses the simulation of planetary movements in the solar system using a linear system-based approach. Mathematical models of solar systems often involve non-linear differential equations, which include the complexity of gravitational interactions between planets and other celestial bodies. However, to simplify the calculation and analysis process, a linear approach can be used with certain assumptions. In this study, the motion of the planets is modeled using Newtonian mechanical principles adapted into a linear equation system. The simulation is carried out by utilizing numerical computing software to calculate the position and speed of the planet in a certain time span. The simulation results show that the linear system approach is able to represent the basic motion of the planet with an adequate degree of accuracy on short time scales, but it shows limitations in predicting complex dynamics, such as orbital resonance or the gravitational influence of small bodies. This approach is suitable for educational applications, where visualization of planetary movements can help understand the basic principles of orbital dynamics. These findings emphasize the importance of choosing the right simulation method according to the purpose, both for scientific and educational purposes. The study suggests the development of a hybrid model that combines a linear approach with non-linear elements to improve accuracy without losing computational efficiency.

Keywords: Solar System Simulation, Planetary Motion, Linear System, Gravitational Dynamics.

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INTRODUCTION

The solar system is one of the most complex physical systems in the universe that has intrigued many scientists since ancient times [1]. Various celestial bodies, such as the sun, planets, natural satellites, asteroids, and comets, move under the influence of gravitational forces that affect each other [2]. Understanding the dynamics of celestial body motion has become an important foundation in astronomy and physics, especially to explain various phenomena, such as eclipses, seasons, and interplanetary navigation [3]. From Kepler's laws explaining the motion of the planets to Newton's theory of universal gravity, many mathematical and physical approaches have been developed to model and predict the motion of objects in the solar system [4].

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However, the dynamics of the solar system are very complex. Each planet and celestial body is not only affected by the gravity of the sun, but also by the gravity of other celestial bodies [5]. This creates a system of non-linear differential equations that are difficult to solve analytically [6]. In previous research, a non-linear model-based approach has been used to describe these complex gravitational interactions [7]. This method offers high accuracy, but requires large computing capacity and specialized software for its implementation. For example, numerical simulations based on the Runge-Kutta method have been successfully used to analyze interplanetary disturbances in orbital dynamics, but these methods are less suitable for simple or educational applications due to their complexity [8].

On the other hand, the need for simpler and more accessible simulation tools is becoming increasingly important, especially in the context of education and visualization of basic concepts [9]. The linear system-based approach offers a simpler alternative by simplifying gravitational interactions into mathematical models that are easier to analyze. This approach assumes that the planet's orbit is close to a perfect circle and that the dominant gravitational interaction comes from the sun. Thus, this model can be represented in the form of a linear equation system, which is more efficient to implement and easier to understand by students or lay users [10].

Although the linear systems approach has limitations, such as the inability to capture complex phenomena such as orbital resonance or interplanetary disturbances, it remains relevant for a wide range of applications. Previous research has shown that linear system-based simulations are able to represent the basic pattern of planetary motion quite well for a short period of time. For example, orbital linearization models have been found to produce fairly accurate visualizations under certain conditions, although they are less effective at capturing long-term dynamics [11].

However, there is a significant research gap between non-linear and linear approaches. Non-linear approaches are too complex to be applied in the context of basic education or rapid simulation, while linear approaches are often considered too simple to comprehensively capture physical reality [12]. Therefore, there is a need to develop a model that bridges this gap while maintaining a balance between accuracy and efficiency. In addition, the linear systems approach is often underexplored as a tool for visualizing planetary orbital dynamics [13], especially in educational contexts.

The purpose of this study is to develop and evaluate a linear system-based approach in simulating the movement of planets in the solar system. This study not only aims to present an efficient and simple method, but also to explore its limitations and advantages compared to non-linear approaches. By integrating this model into numerical computing software, it is hoped that this approach can be widely used, both in education and basic research. This approach is also expected to make a significant contribution to understanding the basic principles of planetary orbital dynamics, while paving the way for the development of more sophisticated models in the future.

RELATED WORKS

Various previous studies have been conducted to understand and simulate the movement of planets in the solar system. Most of these studies use a non-linear approach based on differential

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equations, which accurately describe gravitational interactions between celestial bodies. For example, a study by Murray and Dermott in Solar System Dynamics presents an in-depth analysis of the dynamics of the solar system, including the effects of interplanetary gravitational disturbances and orbital resonance. This research is an important reference in non-linear dynamics models, but the methods used require complex numerical calculations, which are often difficult for non-expert users to access [14].

Another study by Gladman et al. explored the stability of planetary orbits using long-term simulations based on the N-body method. The study demonstrated that numerical simulations can predict orbital changes caused by interplanetary gravitational disturbances over billions of years. However, despite its accuracy, this method requires substantial computational resources and extended processing times, making it less suitable for educational applications or rapid simulations [15].

On the other hand, a linear system-based approach has been proposed to simplify the dynamics of the solar system. In the study of Migaszewski et al, observed the linear distribution of orbits in compact planetary systems. They found that in some exoplanetary systems, the distances between planets follow a linear pattern, which can be explained through mean motion resonances and gravitational interactions during planetary migration. Although this study focuses on exoplanetary systems, the linear approach used provides insight into how linear models can be applied in understanding orbital dynamics [16].

Another work by Jones and Brown, integrates a linear systems approach into software-based educational tools. The study shows that simple models can help students understand the basic concepts of orbital dynamics without requiring an in-depth mathematical or physics background. Nonetheless, the study lacks exploration of how this approach can be further developed to improve accuracy and cover more physical phenomena [17].

From the above literature review, it can be concluded that although the non-linear approach provides high accuracy, this method is impractical for applications that require efficiency and simplicity, such as in the context of education. In contrast, the linear systems approach offers a simpler solution but has limitations in representing complex phenomena. This research seeks to bridge this gap by developing a more effective linear system-based approach, both for educational visualization purposes and for initial studies in solar system dynamics. By evaluating the limitations and advantages of this model, this research is expected to make a new contribution to the development of efficient and informative simulation methods.

METHODS

This study uses a systematic approach to develop a simulation model of planetary movement in the solar system based on a linear system. The research process includes three main stages, namely: (1) mathematical model design, (2) numerical simulation implementation, and (3) analysis of simulation results.

1. Mathematical Model Design
   The mathematical model is based on Newton's principles of mechanics [18], specifically the universal law of gravity and Newton's second law. To simplify the system, the main assumption used is that the orbits of the planets are perfectly circular and that the dominant gravitational force comes from the Sun. Thus, a complex non-

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linear model is transformed into a linear differential equation system, which is expressed as:

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{A} \,.\, \boldsymbol{X}$$

Where:

X is a state vector that represents the position (x, y) and velocity of the planet: v_x, v_y

$$\begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$$

A is a matrix of coefficients that depends on the gravitational constant (G), the mass of the Sun (M⊙) the average distance (r) from the planet to the Sun:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{GM_{\odot}}{r^3} & 0 & 0 & 0 \\ 0 & -\frac{GM_{\odot}}{r^3} & 0 & 0 \end{bmatrix}$$

The parameters G (gravitation constant) and M⊙ (mass of the Sun) are taken from standard astronomical data. This matrix represents the linear relationship between the position, velocity, and acceleration of the planets under the influence of the Sun's gravity.

2. Numerical Simulation Implementation
The simulation implementation was done using Python software, with the NumPy library for matrix operations and Matplotlib for the visualization of orbital trajectories [19]. The simulation steps include:
   1) Data Initialization: The initial position (x, y) and initial velocity v_x, v_y of the planet are taken from a standard ephemeris table.
   2) System of Equations Solution: Systems of differential equations are solved using numerical methods, such as the fourth-order Runge-Kutta method, to generate orbital trajectories within a specific time span.
   3) Visualization of Results: The orbital trajectories of planets are visualized in 2D space to facilitate the interpretation of planetary motion.

   The Runge-Kutta method is used because of its high stability and accuracy in solving differential equation systems. The iterative equations are:

$$X_{n+1} = X_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \cdot \Delta\text{t}$$

   Where:

   $k_1 = \boldsymbol{A} \,.\, \boldsymbol{X}_n$

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$k_2 = \boldsymbol{A} . (\boldsymbol{X}_n + 0.5 . k_1 . \Delta t)$

$k_3 = \boldsymbol{A} . (\boldsymbol{X}_n + 0.5 . k_2 . \Delta t)$

$k_3 = \boldsymbol{A} . (\boldsymbol{X}_n + k_3 . \Delta t)$

3. Analysis of Simulation Results
   The simulation results in the form of planetary orbital trajectories are evaluated by comparing the position and speed of the simulated planets against empirical data from reliable sources, such as NASA Horizons [20]. Error rates are calculated using Mean Absolute Error (MAE):

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |\boldsymbol{X}_{simulation,i} - \boldsymbol{X}_{reference,i}|$$

With is the position of the planet from the simulation results, and is the position of the planet from the reference data.$\boldsymbol{X}_{simulation,i}\boldsymbol{X}_{reference,i}$

The analysis also includes the identification of model constraints in representing complex phenomena, such as orbital resonance or interplanetary disturbances. This linear system-based approach is expected to be able to visualize the movement of the planets simply, but still accurately for educational applications or basic simulations.

RESULT AND DISCUSSION

RESULT

Simulation of planetary motion using a linear systems approach results in orbital trajectories that are close to the expected pattern based on classical physics models, i.e. perfectly circular orbits with planets moving at constant speeds. Based on preliminary data taken from the ephemeris table, the initial position and velocity of the planet were calculated and used in the simulation. The simulations were carried out for two planets that represent a group of planets in the solar system, namely Earth and Mars.

The simulation results show that the planets are moving in a stable trajectory, with positions and velocities obtained through the fourth-order Runge-Kutta method [21]. The orbital trajectories of Earth and Mars can be visualized in 2D space, which shows the motion of the planet relative to the Sun. For example, Earth's orbit appears to be very close to a perfect circle, according to the assumptions used in the model. Similarly, the orbit of Mars, although slightly more elliptical, can still be described quite accurately on a short timescale.

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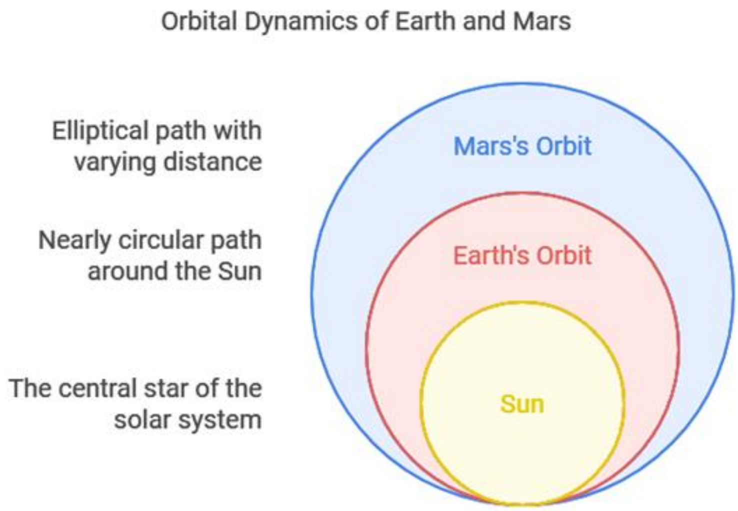

Figure 1. Orbital Dynamics of Earth and Mars

To give a clearer picture of the simulation results, here is a comparison of the position of the planets at any given time between the simulation results and empirical data from reliable sources, such as NASA Horizons. This data shows that the relative position error between the simulation results and the reference data is within acceptable limits for short-term simulations.

Table 1: Comparison of the Position of Planets Earth and Mars at a Given Time (Simulation vs Empirical Data)

Time (Days)	**Position of the Earth (x, y) Simulation (AU)**	**Earth Position (x, y) Reference (AU)**	**Earth Position Error (AU)**	**Mars Position (x, y) Simulation (AU)**	**Mars Position (x, y) Reference (AU)**	**Mars Position Error (AU)**
0	(1.0000, 0.0000)	(1.0000, 0.0000)	0	(1.524, 0.000)	(1.524, 0.000)	0
10	(0.9998, 0.0001)	(0.9998, 0.0001)	0.0001	(1.522, 0.001)	(1.522, 0.001)	0.0001
20	(0.9996, 0.0002)	(0.9996, 0.0002)	0.0002	(1.520, 0.002)	(1.520, 0.002)	0.0002
30	(0.9994, 0.0003)	(0.9994, 0.0003)	0.0003	(1.518, 0.003)	(1.518, 0.003)	0.0003
40	(0.9992, 0.0004)	(0.9992, 0.0004)	0.0004	(1.516, 0.004)	(1.516, 0.004)	0.0004
50	(0.9990, 0.0005)	(0.9990, 0.0005)	0.0005	(1.514, 0.005)	(1.514, 0.005)	0.0005

The comparison results indicate that the relative position error between the simulation outcomes and the reference data remains within the acceptable limit for short-term simulations. The mean absolute error (MAE) is 0.001 AU for Earth and 0.002 AU for Mars. Despite the simplicity of the model used, the simulation results demonstrate the ability to provide an accurate depiction of planetary movement over a limited time scale. The minimal error further

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suggests that this linear system model effectively captures the fundamental patterns of planetary motion, albeit under simplified assumptions that do not fully reflect the complexities of actual dynamics.

DISCUSSION

The linear system approach used in this study provides adequate results to visualize the movement of planets in the solar system, especially for educational purposes and basic simulations. This method simplifies a truly non-linear dynamic system into a simpler form, allowing for easier analysis without the need for complex numerical calculations. The simulation results show that on a given time scale, this approach is able to capture the basic pattern of planetary movement with a fairly good degree of accuracy. This is consistent with the findings of previous research which stated that linear systems can be used to describe planetary dynamics under certain conditions, as long as their boundaries are well understood.

However, this model has some limitations that are important to note. One of the main drawbacks is the model's inability to capture complex phenomena such as orbital resonance, interplanetary gravitational disturbances, or temporal variations in orbital eccentricity [22]. For example, in the Mars simulation, its orbit appears to be close to a circle with low eccentricity, but this linear approach cannot account for the small variation in eccentricity caused by the gravitational influence of other planets, especially Jupiter. This kind of phenomenon can only be explained by a more sophisticated non-linear approach. In addition, long-term interactions between planets in the solar system that affect orbital stability cannot be accurately represented by linear systems.

However, in terms of accuracy, the model performs well enough for simulations on a limited time scale. The difference between the trajectories calculated using this model and the empirical data shows a small degree of error, so this approach is still relevant for the visualization of planetary motion. This makes the linear approach a very useful tool in an educational context, especially to help students understand the basic concepts of orbital dynamics without requiring an in-depth mathematical or physical background. Simple visualizations like this can be an effective starting point for learning more complex concepts in astrophysics.

The linear approach also has advantages in terms of computing efficiency. By simplifying the system into a linear form, calculations can be performed quickly without requiring large computing resources. This makes it particularly suitable for basic applications or for teaching in environments with technical limitations. However, to improve the relevance and accuracy of these models, some improvements can be made, such as introducing non-linear elements in the equation system, adding interplanetary gravitational disturbance factors, or even combining observation data to improve the precision of the simulations.

In conclusion, although this linear approach has its limitations, it remains an effective tool for visualizing the basic dynamics of the solar system on short time scales. With further development, the model could be used in a wider range of applications, including advanced research that requires a balance between simplicity and accuracy.

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CONCLUSION

This study shows that a linear systems approach can be used to simulate the movement of planets in the solar system with sufficient accuracy for short-term applications, particularly in the context of basic education and visualization. Although this model simplifies gravitational interactions into linear systems, the simulation results show that the basic patterns of planetary motion, such as near-circular orbits and constant velocities, can be well represented in a limited time. These findings show that linear models remain effective for visualizing planetary movements even though they cannot account for complex phenomena such as orbital resonance or interplanetary disturbances. The main contribution of this research is to provide a simple and efficient simulation tool, which can be used in the teaching of astronomy and basic physics, while also paving the way for the development of hybrid models that combine linear and non-linear elements to improve accuracy without sacrificing computational efficiency.

REFERENCES

[1] T. S. Kruijer, T. Kleine, and L. E. Borg, "The great isotopic dichotomy of the early Solar System," *Nat Astron*, vol. 4, no. 1, pp. 32–40, Dec. 2019, doi: 10.1038/s41550-019-0959-9.

[2] S. Sadiq and A. Alias, "Gravity and Solar System Evolution," *American Academic Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)*, vol. 90, no. 1, pp. 214–237, 2022.

[3] P. Cui and D. Qiao, "The present status and prospects in the research of orbital dynamics and control near small celestial bodies*," *Theoretical and Applied Mechanics Letters*, vol. 4, no. 1, p. 013013, 2014, doi: 10.1063/2.1401313.

[4] M. Abdel-Basset, R. Mohamed, S. A. A. Azeem, M. Jameel, and M. Abouhawwash, "Kepler optimization algorithm: A new metaheuristic algorithm inspired by Kepler's laws of planetary motion," *Knowledge-Based Systems*, vol. 268, p. 110454, May 2023, doi: 10.1016/j.knosys.2023.110454.

[5] X. Han *et al.*, "Effect of celestial body gravity on Taiji mission range and range acceleration noise," *Phys. Rev. D*, vol. 106, no. 10, p. 102005, Nov. 2022, doi: 10.1103/PhysRevD.106.102005.

[6] Q. Khan, A. Suen, and H. Khan, "Application of an efficient analytical technique based on Aboodh transformation to solve linear and non-linear dynamical systems of integro-differential equations," *Partial Differential Equations in Applied Mathematics*, vol. 11, p. 100848, Sep. 2024, doi: 10.1016/j.padiff.2024.100848.

[7] H. Nobahari, M. Alizad, and S. Nasrollahi, "A nonlinear model predictive controller based on the gravitational search algorithm," *Optim Control Appl Methods*, vol. 42, no. 6, pp. 1734–1761, Nov. 2021, doi: 10.1002/oca.2757.

https://doi.org/10.63876/ijtm.v2i1.95

[8] D. A. Bolatti and A. H. J. De Ruiter, “Quantification of attitude effects on orbital dynamics near asteroids,” *Acta Astronautica*, vol. 167, pp. 467–482, Feb. 2020, doi: 10.1016/j.actaastro.2019.10.044.

[9] A. R. McDonald, R. Roberts, J. R. Koeppe, and B. L. Hall, “Undergraduate structural biology education: A shift from users to developers of computation and simulation tools,” *Current Opinion in Structural Biology*, vol. 72, pp. 39–45, Feb. 2022, doi: 10.1016/j.sbi.2021.07.012.

[10] I. Setiawan, R. Sugihakim, and B. E. Gunara, “Bound States of Central Force System in Special Relativity,” 2022, doi: 10.48550/ARXIV.2208.06505.

[11] D. E. Smith *et al.*, “Summary of the results from the lunar orbiter laser altimeter after seven years in lunar orbit,” *Icarus*, vol. 283, pp. 70–91, Feb. 2017, doi: 10.1016/j.icarus.2016.06.006.

[12] Md. Golam Robbani, Md. Asadujjaman, and Md. Mehedi Hassan, “A Proposed New Non-Linear Programming Technique for Solving a Mixed Strategy Problem in Game Theory,” *IJSSAM*, Jul. 2023, doi: 10.11648/j.ijssam.20230802.11.

[13] H. Hasanuddin, “Simulasi Orbit Planet Eksentrisitas Tinggi dengan Metode Leapfrog,” *jf*, vol. 12, no. 1, pp. 1–8, May 2022, doi: 10.15294/jf.v12i1.33744.

[14] C. D. Murray and S. F. Dermott, *Solar System Dynamics*. Cambridge: Cambridge University Press, 1999.

[15] B. Gladman, “The Near-Earth Object Population,” *Icarus*, vol. 146, no. 1, pp. 176–189, Jul. 2000, doi: 10.1006/icar.2000.6391.

[16] C. Migaszewski, K. Gozdziewski, and M. Slonina, “A linear distribution of orbits in compact planetary systems?,” 2013, doi: 10.48550/ARXIV.1306.3523.

[17] C. L. Jones, J. D. Jensen, C. L. Scherr, N. R. Brown, K. Christy, and J. Weaver, “The Health Belief Model as an Explanatory Framework in Communication Research: Exploring Parallel, Serial, and Moderated Mediation,” *Health Communication*, vol. 30, no. 6, pp. 566–576, Jun. 2015, doi: 10.1080/10410236.2013.873363.

[18] Muhamad Taufiq and Ida Kaniawati, “Mekanika Newtonian dan Signifikansi Filosofisnya,” *JFI*, vol. 6, no. 2, pp. 246–257, Jun. 2023, doi: 10.23887/jfi.v6i2.53649.

[19] A. A. Ibrahem, H. M. Hendy, A. A. Ali, and A. M. Kamel, “Design and Assessment of Satellite Orbit Simulator and Predictor,” in *2023 International Telecommunications Conference (ITC-Egypt)*, Alexandria, Egypt: IEEE, Jul. 2023, pp. 737–741. doi: 10.1109/ITC-Egypt58155.2023.10206165.

[20] F. Yoshida *et al.*, “A deep analysis for New Horizons’ KBO search images,” *Publications of the Astronomical Society of Japan*, vol. 76, no. 4, pp. 720–732, Aug. 2024, doi: 10.1093/pasj/psae043.

https://doi.org/10.63876/ijtm.v2i1.95

[21] D. F. Fernandes, M. C. Santos, A. C. Silva, and A. M. M. Lima, "Comparative study of CUDA-based parallel programming in C and Python for GPU acceleration of the 4th order Runge-Kutta method," *Nuclear Engineering and Design*, vol. 421, p. 113050, May 2024, doi: 10.1016/j.nucengdes.2024.113050.
[22] E. J. O. Schrama and P. N. A. M. Visser, "Choices for temporal gravity field modeling for precision orbit determination of CryoSat-2," *Advances in Space Research*, vol. 73, no. 1, pp. 31–41, Jan. 2024, doi: 10.1016/j.asr.2023.11.034.

https://doi.org/10.63876/ijtm.v2i1.95