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Population Dynamics Modeling with Differential Equation Method

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Abstract: Population dynamics modeling is one of the important approaches in understanding population development and its influence on various aspects of life, such as economic, social, and environmental. This article discusses the application of differential equation methods in modeling population dynamics, with a focus on the analysis of growth and interactions between populations. The models used include exponential growth models, logistics, and the Lotka-Volterra model to describe competitive interactions and predations between populations. Through numerical simulations and qualitative analysis, this article shows how parameters such as birth rate, mortality, and environmental carrying capacity affect population growth patterns. In addition, the influence of external factors such as government policies and natural disasters is also incorporated into the model to expand the application in real contexts. The results of the analysis show that the differential equation model is able to provide an accurate picture of population dynamics if the parameters are estimated correctly. This article also highlights the importance of model validation using empirical data to ensure prediction reliability. This modeling can be used as a tool in development planning, resource allocation, and risk mitigation in various sectors. The conclusion of this study is that the differential equation method is not only effective in explaining population phenomena, but also flexible to adapt to various dynamic conditions. As such, this approach offers a significant contribution to demographic studies and data-driven decision-making.

Keywords: Population dynamics modeling, differential equations, population growth, interpopulation interactions, numerical simulations.

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INTRODUCTION

Population dynamics is one of the important aspects in demographic studies related to changes in the number, structure, and distribution of the population over time [1]. These changes are

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influenced by various factors, such as birth rates, mortality, migration, and socio-economic policies. A deep understanding of population dynamics is needed, especially for development planning, resource management, and mitigation of environmental impacts due to population growth [2].

In the study of population dynamics, the mathematical approach through modeling becomes a very useful tool. One of the methods that is often used is the differential equation method, which is able to describe population change as a function of time [3]. This method allows for a quantitative analysis of the factors that affect growth and interactions between populations, both in the context of ecosystems and human societies. The developed model can provide insights into how populations are growing, stabilizing, or declining under various conditions [4].

Exponential growth and logistical models are two basic examples of population modeling using differential equations. The exponential model describes unlimited growth, while the logistics model takes into account the supporting capacity of the environment that limits growth [5]. In addition, interactions between populations, such as competition, predation, and symbiosis, can be modeled using the Lotka-Volterra approach [6]. This modeling is not only relevant for theoretical analysis, but also has practical applications in urban planning, agriculture, and biodiversity conservation.

The use of differential equations in population dynamics modeling requires a deep understanding of the parameters and assumptions used [7]. Validating the model with empirical data is an important step to ensure the accuracy and relevance of the results produced. In addition, the integration of external factors, such as climate change and policy interventions, can increase the complexity of the model, but also expand its application in real contexts [8].

This article aims to discuss the application of the differential equation method in modeling population dynamics. Through theoretical analysis and numerical simulations, it is hoped that readers can understand how these methods are used to predict and manage population changes. This article also highlights the potential of differential equation modeling in making a significant contribution to demographic studies and data-driven planning.

RELATED WORKS

Population dynamics modeling has become a widely discussed topic in the scientific literature, with a variety of mathematical and statistical approaches being used to describe population growth patterns and interactions between individuals or groups [9]. One of the earliest approaches introduced was the exponential growth model by Thomas Malthus in the 18th century. In this model, the population is assumed to grow exponentially indefinitely, which is very useful for understanding population growth in its early stages [10]. However, this model does not take into account the carrying capacity of the environment, making it less relevant in the long run [11].

The logistics model introduced by Pierre-François Verhulst in the 19th century overcomes these limitations by including limiting factors in the form of environmental carrying capacity [12]. This model is often used to describe populations that grow to equilibrium due to resource

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limitations. Further research developed this model by taking into account other factors, such as environmental variations and external influences, to improve prediction accuracy [13].

In addition to individual growth models, interactions between populations are also the focus of research. The Lotka-Volterra model is one of the most well-known approaches to modeling predator-prey dynamics or interspecies competition [14]. This model is used extensively in ecology to understand complex interactions between populations and their impact on ecosystems [15]. Several studies have developed this model by including additional factors, such as migration, habitat change, and human impact.

A partial differential equation-based approach is also used to model the spatial distribution of populations [16]. For example, the Fisher-KPP model[17] and diffusion reaction have been applied to describe the dispersal of populations in space and time [18]. This approach has become very relevant in the study of epidemiology and human migration.

In the context of human populations, various studies have utilized differential equation models to project demographic changes [19]. For example, research related to population growth in urban areas often uses logistics models with varying parameters that reflect social and economic dynamics. Some studies also integrate government policies, such as birth control or urbanization, into models to estimate their impact on population growth.

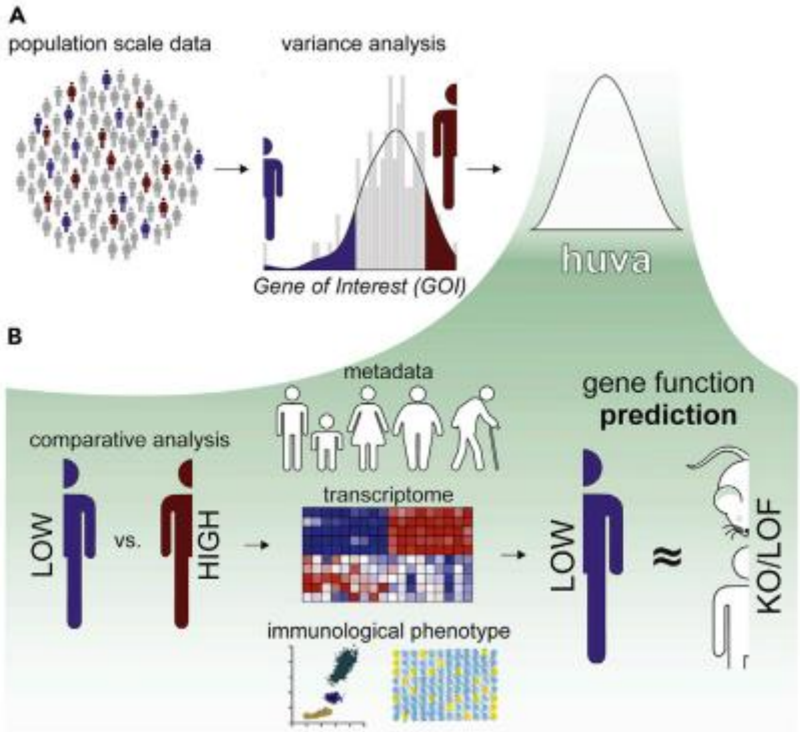

Figure 1. Huva allows comprehensive human variation analysis from several cell types[17]

Numerical simulations play an important role in modeling population dynamics [20]. With the advancement of computing technology, many studies are adopting numerical methods to solve complex nonlinear differential equations [21]. This allows for the exploration of various population growth scenarios, including the effects of climate change, natural disasters, and shifting migration patterns.

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This study shows that the differential equation method has made a significant contribution in understanding population dynamics. This article continues these efforts by exploring the application of this model to solve contemporary demographic problems and proposing a more integrated approach to model population dynamics.

METHODS

The approach in this study uses the differential equation method to model population dynamics. The model developed includes basic population growth (exponential and logistical) as well as interpopulation interactions using the Lotka-Volterra model. Each model is simulated numerically to explore the influence of key parameters on population change patterns.

The exponential growth model is used as a first step to understanding population dynamics without environmental constraints. This model is expressed by differential equations:

$$\frac{dP}{dt} = rP$$

where is the number of population, is the growth rate, and is the time. The solution of this equation gives an idea of the exponential growth where is the initial population.$PrtP(t) = P_0 e^{rt} P_0$

To illustrate the limitations of the environment, a logistics model was used. This model takes into account the environmental carrying capacity with the equation:(K)

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

This model results in a growth pattern that approaches the maximum capacity of the population as time increases.(t)

The dynamics of interactions between populations were analyzed using the Lotka-Volterra model, which represents predator-prey interactions. The model consists of two differential equations:

$$\frac{dN}{dt} = \alpha N - \beta NP$$

$$\frac{dP}{dt} = \delta NP - \gamma P$$

where is the prey population, is the predator population, is the growth rate of prey, is the consumption rate of predators to prey, is the efficiency of energy conversion of prey into predator population, and is the mortality rate of predators.$NP\alpha\beta\delta\gamma$

Numerical simulations are performed using the Euler method to solve differential equations. The parameters used are set to represent various scenarios, such as populations with high carrying capacity, populations with external interventions, and the effects of disasters. Model validation is carried out by comparing the simulation results with empirical data to ensure their accuracy and relevance.

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This approach allows for the exploration of different aspects of population dynamics, from simple population growth to complex interactions, in order to provide insight into patterns of population change under various environmental and social conditions.

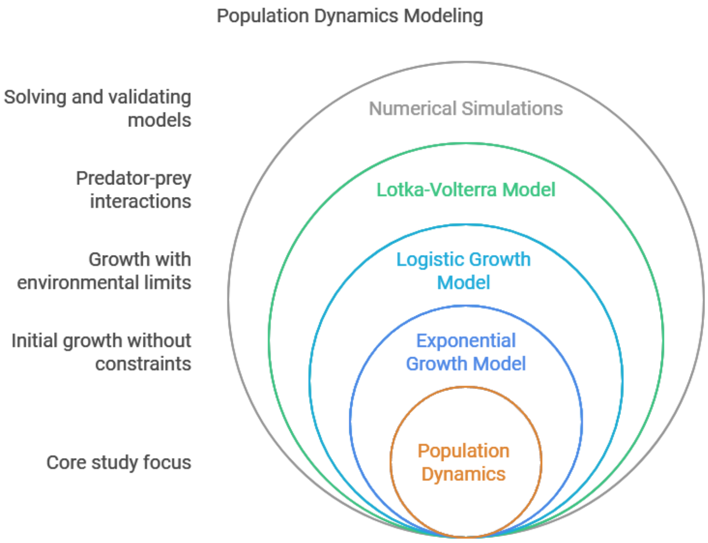

Figure 2. Population Dynamics Modeling

RESULT AND DISCUSSION

The results of the study show different population growth patterns based on the models used. In the exponential growth model, the population grows continuously indefinitely. To support the quantitative analysis, the simulation results of each model are summarized in the following table. This table presents the initial and final population values for each model, providing an overview of the growth patterns and population interactions that occurred during the simulation period.

Table 1: Table of simulation results

Type	Initial Population	Final Population
Exponential Growth	10	200.86
Logistic Growth	10	69.06
Lotka-Volterra (Prey)	40	10.49
Lotka-Volterra (Predator)	9	23.83

With an initial population of 10, the total population reached about 200.86 at the end of the simulation time. This model is relevant for populations that do not face environmental constraints, such as in the early stages of colonization of new territories.

The logistics model, which takes into account the carrying capacity of the environment, shows that the population grows to close to the threshold value of capacity (69.06). This pattern

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reflects a real-life situation where resource availability limits population growth, making it more realistic compared to exponential models for the long term.

In the Lotka-Volterra model, the simulation results show complex interactions between predator and prey populations. The prey population fluctuated from 40 to declining to close to 10.49, while the predator population increased from 9 to 23.83 at the end of the simulation. The graph shows the oscillation of predator-prey populations, illustrating the dynamic dependence between the two. This is in accordance with the pattern of ecological interactions that are often found in real systems.

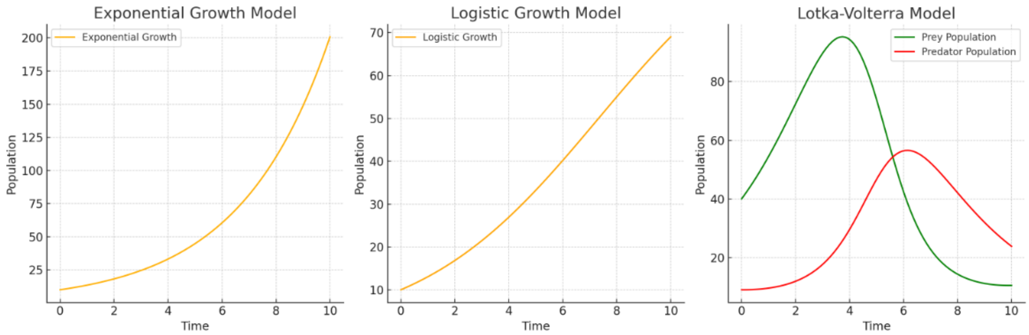

Figure 3: Complex interactions between predator and prey populations

The graph above provides a visual representation for each model. The first graph shows exponential growth, the second graph shows logistical growth, and the third graph shows predator-prey fluctuations in the Lotka-Volterra model.

CONCLUSION

This article has discussed the modeling of population dynamics using differential equation methods, including exponential growth, logistics, and Lotka-Volterra models. The results show that each model has the advantage of describing certain aspects of population dynamics. Exponential models are effective for understanding unlimited population growth, while logistics models offer a more realistic picture by taking into account the carrying capacity of the environment. Meanwhile, the Lotka-Volterra model managed to represent the dynamic interaction between predators and prey, showing the oscillations of populations typical of ecological systems. The numerical simulations carried out confirmed that the differential equation method is able to provide deep insights into population change patterns, both in simple and complex contexts. In addition, this approach is flexible to integrate external factors such as policy, environmental change, and migration, which are relevant in contemporary demographic studies. This modeling has a wide range of application potential, including in development planning, resource management, biodiversity conservation, and risk mitigation due to population growth. It is important to note that the accuracy of the model is highly dependent on parameter estimation and validation with empirical data. Further steps, such as comprehensive data collection and the development of more integrated models, are needed to improve the reliability and relevance of the results. The differential equation method is not only

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a theoretical analysis tool, but also supports data-based decision-making in managing population dynamics effectively and sustainably. This research contributes to understanding the basic mechanisms of population change and proposes mathematical approaches that can be adapted to various conditions and needs.

REFERENCES

[1] L. De Cubber, S. Lefebvre, T. Lancelot, D. S. F. Jorge, and S. M. Gaudron, "Unravelling mechanisms behind population dynamics, biological traits and latitudinal distribution in two benthic ecosystem engineers: A modelling approach," *Prog. Oceanogr.*, vol. 219, p. 103154, Dec. 2023, doi: 10.1016/j.pocean.2023.103154.

[2] M. Song, X. Gong, J. Du, T. Lu, and R. J. Jiao, "Population dynamics modeling of crowdsourcing as an evolutionary Cooperation-Competition game for fulfillment capacity balancing and optimization of smart manufacturing services," *Comput. Ind. Eng.*, vol. 197, p. 110572, Nov. 2024, doi: 10.1016/j.cie.2024.110572.

[3] G. Zeng, S. Chen, H. Wu, and M. Yang, "Continuous-discrete extended Kalman filtering based on the neural ordinary differential equations method," *Eng. Appl. Artif. Intell.*, vol. 133, p. 108611, Jul. 2024, doi: 10.1016/j.engappai.2024.108611.

[4] S. S M, P. Kumar, and V. Govindaraj, "A neural networks-based numerical method for the generalized Caputo-type fractional differential equations," *Math. Comput. Simul.*, vol. 213, pp. 302–323, Nov. 2023, doi: 10.1016/j.matcom.2023.06.012.

[5] S. L. Manu and S. Shikaa, "Mathematical modeling of Taraba State population growth using exponential and logistic models," *Results Control Optim.*, vol. 12, p. 100265, Sep. 2023, doi: 10.1016/j.rico.2023.100265.

[6] Z. Li *et al.*, "Individual dose recommendations for drug interaction between tacrolimus and voriconazole in adult liver transplant recipients: A semiphysiologically based population pharmacokinetic modeling approach," *Eur. J. Pharm. Sci.*, vol. 184, p. 106405, May 2023, doi: 10.1016/j.ejps.2023.106405.

[7] T. V. San Miguel, D. Da Re, and V. Andreo, "A systematic review of Aedes aegypti population dynamics models based on differential equations," *Acta Trop.*, vol. 260, p. 107459, Dec. 2024, doi: 10.1016/j.actatropica.2024.107459.

[8] A. Shahjalal, "Validation and modification of a semi-empirical model for sound absorption by perforated liners in the absence of flow based on comparisons with data from full scale measurements," *Fuel Commun.*, vol. 14, p. 100087, Mar. 2023, doi: 10.1016/j.jfueco.2023.100087.

[9] M. N. Maunder, "Towards a comprehensive framework for providing management advice from statistical inference using population dynamics models," *Ecol. Modell.*, vol. 498, p. 110836, Dec. 2024, doi: 10.1016/j.ecolmodel.2024.110836.

[10] J. S. Cánovas and M. Muñoz-Guillermo, "On the dynamics of a hyperbolic–exponential model of growth with density dependence," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 104, p. 106050, Jan. 2022, doi: 10.1016/j.cnsns.2021.106050.

[11] P. Palumbo, S. Busti, and M. Vanoni, "A cell growth model accounting for both ribosomal sub-units synthesis: conditions for the exponential growth," *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 7240–7245, 2023, doi: 10.1016/j.ifacol.2023.10.332.

https://doi.org/10.63876/ijtm.v1i3.107

[12] M. M. Islam and M. Arakawa, “Development of an integrated scenario-based stochastic rolling-planning multistage logistics model considering various risks,” *Heliyon*, vol. 9, no. 11, p. e22289, Nov. 2023, doi: 10.1016/j.heliyon.2023.e22289.

[13] U. N. Gogoi, P. Saikia, L. Devi, L. Khataniar, and D. J. Mahanta, “Rapid parameter estimation of modified Gompertz and Logistic model for analyzing the growth of Escherichia Coli K2,” *Int. J. Thermofluids*, vol. 24, p. 100851, Nov. 2024, doi: 10.1016/j.ijft.2024.100851.

[14] A. V. Platonov, “Analysis of the dynamical behavior of solutions for a class of hybrid generalized Lotka–Volterra models,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 119, p. 107068, May 2023, doi: 10.1016/j.cnsns.2022.107068.

[15] A. L. A. de Araujo, A. C. Fassoni, K. F. L. Madalena, and L. F. Salvino, “Analysis and simulation of an integro-differential Lotka–Volterra model with variable reproduction rates and optimal control,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 134, p. 108000, Jul. 2024, doi: 10.1016/j.cnsns.2024.108000.

[16] R. Nuca, E. Storvik, F. A. Radu, and M. Icardi, “Splitting schemes for coupled differential equations: Block Schur-based approaches & Partial Jacobi approximation,” *Comput. Math. with Appl.*, vol. 161, pp. 190–201, May 2024, doi: 10.1016/j.camwa.2024.02.042.

[17] G. C. Paul, Tauhida, and D. Kumar, “Revisiting Fisher-KPP model to interpret the spatial spreading of invasive cell population in biology,” *Heliyon*, vol. 8, no. 10, p. e10773, Oct. 2022, doi: 10.1016/j.heliyon.2022.e10773.

[18] T. V. Gortsas, S. V. Tsinopoulos, and D. Polyzos, “A local domain boundary element method for solving the nonlinear fisher KPP diffusion-reaction equation,” *Eng. Anal. Bound. Elem.*, vol. 138, pp. 177–188, May 2022, doi: 10.1016/j.enganabound.2022.02.008.

[19] B. Castillo, M. T. Muñoz-Quezada, and C. Sapiente-Aguirre, “Demographics and tenure of the Chilean urban dog population. A mathematical model,” *Prev. Vet. Med.*, vol. 225, p. 106141, Apr. 2024, doi: 10.1016/j.prevetmed.2024.106141.

[20] I. Ghosh, M. M. Rashid, and S. Mawa, “An evaluation of multispecies population dynamics models through numerical simulations using the new iterative method,” *Healthc. Anal.*, vol. 4, p. 100283, Dec. 2023, doi: 10.1016/j.health.2023.100283.

[21] M. Mahbub and R. M. Shubair, “Contemporary advances in multi-access edge computing: A survey of fundamentals, architecture, technologies, deployment cases, security, challenges, and directions,” *J. Netw. Comput. Appl.*, vol. 219, p. 103726, Oct. 2023, doi: 10.1016/j.jnca.2023.103726.

https://doi.org/10.63876/ijtm.v1i3.107